# 1 Is this a mathematical argument?

## 1.1 Example One

A: If our examples are like this, it is easy.

B: Yes. Can we do it if our examples are little bit different?

C: Say we have an example like this. In this case the preconditions are satisfied, but $X$ happens and then $Y$ and $Z$ and we don't get the predicted answer. I must be missing something here

D: This isnt true – $W$ will happen.

B: No, youre not right. $W$ will happen, in the following way.

C: Ohhh I misunderstood the problem. I saw $P$ as $Q$, in which case we would get $Z$. But apparently it means $R$, so that it is possible for $V$ to happen. Got it.

## 1.2 Example Two

A: Imagine an x with property P1. This x falsifies the proposition that X implies Y, because when we do S1, x will not have property P2. Nor will it help to do S2. Besides, for each part of x we have V1, so x is not Y.

B: Good show. Let us call it Counterexample 1. Now what?

C: Sir, your composure me. A single counterexample refutes a claim as effectively as ten. The claim and its argument have completely misfired. Hands up! You have to surrender. Scrap the false claim, forget about it and try a radically new approach.

B: I agree with you that the claim has received a severe criticism by A's counterexample. But it is untrue that the argument has completely misfired. If, for the time being, you agree to my earlier proposal to use the word 'argument' for a thought-experiment which leads to decomposition of the original claim into subclaims instead of using it in the sense of a guarantee of certain truth, you need not draw this conclusion. My argument certainly showed the claim in the first sense, but not necessarily in the second. You are interested only in arguments which argue what they have set out to do. I am interested in arguments even if they do not accomplish their intended task. Columbus did not reach India but he discovered something quite interesting.

A: So according to your philosophy - while a counterexample to a premise (if it is not a counterexample to the claim at the same time) is a criticism of the argument, but not of the claim - a counterexample to the claim is a criticism of the claim, but not necessarily of the argument. You agree to surrender as regards the claim, but you defend the argument. But if the claim is false, what on earth does the argument show?

C: Your analogy with Columbus breaks down. Accepting a counterexample to the claim must mean total surrender.

D: But why accept the counterexample? We shown our claim - now it is a fact. I admit that it clashes with this so-called ‘counterexample’ One of them has to give way. But why should the fact give way, when it has been shown? It is the criticism that should retreat. It is fake criticism. This x is not an X. It is a monster, a pathological case, not a counterexample.

C: Why not? An X is a D. And x is a D1.

B: Let us call this definition Def. 1.

D: Your definition is incorrect. An X is a D2.

B: Call this Def. 2.

### 1.3 Example Three

A: For all X’s, Y holds, due to Z.

B: But this is not an X, it’s a W

A: No it’s not, it doesn’t have property P1

B: We do S1

A: Also it doesn’t have propery P2, since it is S2

B: It is S2, because of S3.

## 2 Analysing mathematical argument

**Haim: 5.** The following reformulation of the problem may be useful: Show that for any permutation s in Sn, the sum $a_s(1) + a_s(2) + a_s(j)$ is not in M for any $j =< n$.

Now, we may use the fact that Sn is “quite large” and prove the existence of such permutation with some kind of a pigeonhole-ish principle

20 July, 2009 at 6:51 am

**Nate: 7.** Well, my first thought is to see if the hypotheses seem reasonable. The hypothesis that $s = a_1 + \; + a_n$ not lie in M is certainly necessary, as the last jump that the grasshopper takes will land on s. The grasshoppers other steps will land on a partial sums $a_{\sigma(1)} + \; + a_{\sigma(k)}$ for some permutation $\sigma$, but we get to choose the permutation. Thus it seems plausible that we can avoid a given set of n-1 points.

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**Thomas: 8.** Quick observation. The grasshopper must make a first step. This is always possible, since the $a_i$ are distinct and $|M| = n - 1$; that is, there is always an $a_i$ not in M. However, let’s say M matches all but one of the $a_i$. Then the first step is uniquely determined. Still, according to the claimed theorem, a second step must still be possible.

20 July, 2009 at 7:00 am

**Haim: 9.** Following (3); For any x in M, there are two possibilities: 1. x cant be represented as a sum of (distinct) ai's. 2. $x = a_j 1 + a_j 2 + a_j k$. In this case, we may assign x the set j1, j2jk

So M can actually be regarded as a subset of P(1, 2n)

20 July, 2009 at 7:01 am

**Dave: 10.**

Addressing Haim(5):

Thats pretty strong; all you need is that there exists a permutation where that is true. And it doesnt work; there are numbers $a_1, a_2, \ldots, a_n$ and sets $M$ of $n-1$ points such that, for instance, $a_1 \in M$. Then any permutation starting with $a_1$ would not satisfy your conjecture for $j = 1$.

But, just looking for *one* permutation that satisfies $a_s(1) + a_s(2) + a_s(j) \notin M$ for any $j \leq n$ (which is basically the statement of the theorem), could lend itself well to induction. In other words, use the fact that for every subset $M \subset M$ of size $j$ not containing $a_s(1) + a_s(2) + a_s(j)$, there is a way to permute those $j$ numbers to avoid $M$.