

# 1 Is this a mathematical argument?

## 1.1 Example One

[extract from <http://polymathprojects.org/2011/07/19/minipolymath3-project-2011-imo/>, thread 3.]

A: If the points form a convex polygon, it is easy.

B: Yes. Can we do it if there is a single point not on the convex hull of the points?

C: Say there are four points: an equilateral triangle, and then one point in the center of the triangle. No three points are collinear. It seems to me that the windmill can not use the center point more than once! As soon as it hits one of the corner points, it will cycle indefinitely through the corners and never return to the center point. I must be missing something here

D: This isnt true it will alternate between the centre and each vertex of the triangle.

B: No, youre not right. Let the corner points be A, B, C, clockwise, M the center. If you start in M, you first hit say A, then C, then M, then B, then A.

C: Ohhh... I misunderstood the problem. I saw it as a half-line extending out from the last point, in which case you would get stuck on the convex hull. But apparently it means a full line, so that the next point can be "behind" the previous point. Got it.

## 1.2 Example Two

[Extract from Lakatos, Proofs and Refutations, pp14-16]

ALPHA: Imagine a solid bounded by a pair of nested cubes - a pair of cubes, one of which is inside, but does not touch the other (Fig. 5). This hollow cube falsifies your first lemma, because on removing a face from the inner cube, the polyhedron will not be stretchable on to a plane. Nor will it help to remove a face from the outer cube instead. Besides, for each cube  $V - E + F = 2$ , so that for the hollow cube  $V - E + F = 4$ .

TEACHER: Good show. Let us call it Counterexample 1. Now what?

GAMMA: Sir, your composure me. A single counterexample refutes a conjecture as effectively as ten. The conjecture and its proof have completely misfired. Hands up! You have to surrender. Scrap the false conjecture, forget about it and try a radically new approach.

TEACHER: I agree with you that the conjecture has received a severe criticism by Alpha's counterexample. But it is untrue that the proof has completely misfired. If, for the time being, you agree to my earlier proposal to use the word 'proof' for a thought-experiment which leads to decomposition of the original conjecture into subconjectures instead of using it in the sense of a guarantee of certain truth, you need not draw this

conclusion. My proof certainly proved Eulers conjecture in the first sense, but not necessarily in the second. You are interested only in proofs which prove what they have set out to prove. I am interested in proofs even if they do not accomplish their intended task. Columbus did not reach India but he discovered something quite interesting.

ALPHA: So according to your philosophy - while a local counterexample (if it is not global at the same time) is a criticism of the proof, but not of the conjecture - a global counterexample is a criticism of the conjecture, but not necessarily of the proof. You agree to surrender as regards the conjecture, but you defend the proof. But if the conjecture is false, what on earth does the proof prove?

GAMMA: Your analogy with Columbus breaks down. Accepting a global counterexample must mean total surrender.

DELTA: But why accept the counterexample? We proved our conjecture - now it is a theorem. I admit that it clashes with this so-called 'counterexample' One of them has to give way. But why should the theorem give way, when it has been proved? It is the criticism that should retreat. It is fake criticism. This pair of nested cubes is not a polyhedron at all. It is a monster, a pathological case, not a counterexample.

GAMMA: Why not? A polyhedron is a solid whose surface consists of polygonal faces. And my counterexample is a solid bounded by polygonal faces.

TEACHER: Let us call this definition Def. 1.

DELTA: Your definition is incorrect. A polyhedron must be a surface: it has faces, edges, vertices, it can be deformed, stretched out on a blackboard, and has nothing to do with the concept of solid. 'A polyhedron is a surface consisting of a system of polygons'.

TEACHER: Call this Def. 2.

### 1.3 Example Three

A: Shops are not allowed to open on Sunday, due to the 1977 Worker Protection and Working Environment Act.

B: But this is not a shop, it's a petrol station

A: No it's not, it doesn't sell petrol

B: We cater to motorists by selling products and accessories for cars, such as cans of oil and fluffy dice

A: Also this is too big to be a petrol station since it is bigger than 100 square metres

B: Our premises are bigger than 100 square metres but we have cordoned off part of it, so there is only a space of 100 square metres which is open on Sundays.